Agricultural Production and Technological Change

Advanced Producer Theory and Analysis: Equilibrium Displacement Models

Alexandra E. Hill

AREC 705: Advanced Market Models

Colorado State University

What are the uses of EDMs?

- They provide insight into the quantitative importance of influences contained in an equilibrium system.
- They are useful for forecasting and policy evaluation.
- They are useful for analyzing the welfare implications of policies along the supply chain.
- Effects of shifts in supply and demand on **food price spreads** (Gardner 1975); **nutrition effects** (Perrin and Scobie 1981); **food production** (Rutledge and Merel 2022)

What are EDMs?

- Partial equilibrium, multi-market models
- They assert a vertical (and sometimes horizontal) structure for incidence
- They consist of logarithmic differential equations representing changes in supply and demand
- They require estimates of supply and demand elasticities when used in forecasting exercises (and cross-elasticities when considering multiple, related markets)

The simplest model is one that describes a single good in one market.

In this case, we can represent the market in terms of supply and demand (generically) as:

$$Q^s = f(P, z_1, z_2, \dots\ z_n)$$

$$Q^d = g(P, y_1, y_2, \dots\ y_n)$$

where each z_i represents an exogenous supply shifter (e.g., a tax) and each y_i represents an exogenous demand shifter (e.g., advt.).

We can *never* know the true functional forms of supply and demand equations.

However, we often assume a functional form (e.g., Cobb-Douglas).

The alternative (what we will do here) is to linearly approximate supply and demand locally and use the log approximation, i.e. that $d\ln(X) \approx \frac{dX}{X}$.

Note that this approximation is reliably accurate when changes in equilibrium are “small”

Buse (1958) introduced "total elasticity" estimated by taking the total derivatives of supply (and demand):

$$dQ^s = \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial z_1}dz_1 + \frac{\partial f}{\partial z_2}dz_2 + \ldots + \frac{\partial f}{\partial z_n}dz_n$$

Which we can transform to:

$$\frac{dQ^s}{Q} = \frac{\partial f}{\partial P}\frac{P}{Q}\frac{dP}{P} + \frac{\partial f}{\partial z_1}\frac{z_1}{Q}\frac{dz_1}{z_1} + \frac{\partial f}{\partial z_2}\frac{z_2}{Q}\frac{dz_2}{z_2} + \ldots + \frac{\partial f}{\partial z_n}\frac{z_n}{Q}\frac{dz_n}{z_n}$$

And we can write this in elasticity form as:

$$\ln(Q^s) = \epsilon\ d\ln(P) + \beta$$

$$\ln(Q^d) = \eta\ d\ln(P) + \alpha$$

where β and α represent the terms for supply and demand shifters, respectively.

If we have only one supply shifter and one demand shifter, we can define them as:

$$\beta = \frac{\partial f}{\partial z_1} \frac{z_1}{Q^s} \frac{dz_1}{z_1} = \epsilon_z \; d\ln(z_1)$$
$$\alpha = \frac{\partial g}{\partial y_1} \frac{y_1}{Q^d} \frac{dy_1}{y_1} = \eta_y \; d\ln(y_1)$$

β can be thought of as something that increases supply, like technological change or policies, and α representing something that increases demand, like advertising or a change in income.

We can solve this system of simultaneous equations by setting them equal to one another:

$$Q^s = Q^d \Rightarrow \epsilon \ d \ln(P) + \beta = \ln(Q^d) = \eta \ d \ln(P) + \alpha$$

$$\Rightarrow \ln(P) = \frac{\alpha - \beta}{\epsilon - \eta}$$

Or we can write the equations in terms of vertical shifters:

$$\ln P = \frac{1}{\epsilon} \ln Q - b, \text{ where } b = \frac{1}{\epsilon}\beta$$

$$\ln P = \frac{1}{\eta} \ln Q - a, \text{ where } a = \frac{1}{\eta}\alpha$$

EDMs – Notation

Note that (as in Wohlgenant 2012) these equations are commonly expressed in the "E" format where E denotes relative change so that $EQ = \frac{\Delta Q}{Q} \approx \Delta \ln Q$:

$$EQ_S = \varepsilon EP - \varepsilon k$$

$$EQ_D = \eta EP - \eta \delta$$

$$\text{setting } EQ_S = EQ_D \text{ and yielding } EP = \frac{\varepsilon k - \eta \delta}{\varepsilon - \eta}$$

EDMs are often used to speak to welfare implications from shifts in supply and demand:

$$\Delta CS = -(\Delta P - \delta P_0)(Q_0 + 0.5\Delta Q) = -P_0 Q_0 (EP - \delta)(1 + 0.5EQ)$$

$$\Delta PS = -(\Delta P - kP_0)(Q_0 + 0.5\Delta Q) = -P_0 Q_0 (EP - k)(1 + 0.5EQ)$$

This assumes that either supply and demand curves are linear or can be approximated by linear functions in the neighborhood of the two equilibrium points.

These estimate changes from parallel shifts in supply and demand.

EDMs – Outcomes of Interest

Perhaps a more reasonable assumption is that demand and supply elasticities are constant (but not necessarily that supply and demand curves are linear):

$$\Delta CS = -\left(1+\eta\right)^{-1} P_0 Q_0 \left(e^{(1+\eta)EP-\eta\delta} - 1\right)$$

$$\Delta PS = -\left(1+\varepsilon\right)^{-1} P_0 Q_0 \left(e^{(1+\varepsilon)EP-\varepsilon k} - 1\right)$$

These estimate changes from proportional shifts in supply and demand.

Perhaps a more reasonable assumption is that demand and supply elasticities are constant (but not necessarily that supply and demand curves are linear):

$$\Delta CS = -\left(1+\eta\right)^{-1} P_0 Q_0 \left(e^{(1+\eta)EP-\eta\delta} - 1\right)$$

$$\Delta PS = -\left(1+\varepsilon\right)^{-1} P_0 Q_0 \left(e^{(1+\varepsilon)EP-\varepsilon k} - 1\right)$$

These estimate changes from proportional shifts in supply and demand.

EDMs – An Application to Taxes

Consider the following supply and demand equations:

$$Q^d = Q^d(P^d)$$

$$Q^s = Q^s(P^s)$$

$$Q^d = Q^s$$

$$P^d = (1 + t)P^s$$

EDMs – Two Related Commodities

We can totally differentiate and apply the same transformations as in the single market case to arrive at equations for proportional changes:

$$d \ln Q_1^s = \eta_{1,1} d \ln P_1^d + \eta_{1,2} d \ln P_1^d + \alpha_1$$
$$d \ln Q_2^s = \eta_{2,1} d \ln P_1^d + \eta_{2,2} d \ln P_2^d + \alpha_2$$
$$d \ln Q_1^d = \epsilon_{11} d \ln P_1^s + \epsilon_{12} d \ln P_2^s + \beta_1$$
$$d \ln Q_2^d = \epsilon_{21} d \ln P_1^s + \epsilon_{22} d \ln P_2^s + \beta_2$$

Then we can set $Q_1^s = Q_1^d$ and $Q_2^s = Q_2^d$

EDMs – Two Related Commodities

As we add more equations, this system becomes simpler to solve with matrix algebra:

$$\begin{bmatrix} 1 & 0 & -\eta_{11} & -\eta_{12} \\ 0 & 1 & -\eta_{21} & -\eta_{22} \\ 1 & 0 & -\epsilon_{11} & -\epsilon_{12} \\ 0 & 1 & -\epsilon_{21} & -\epsilon_{22} \end{bmatrix} \begin{bmatrix} dlnQ_1 \\ dlnQ_2 \\ dlnP_1 \\ dlnP_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

EDMs – Two Related Commodities

Rearranging, we have:

$$\begin{bmatrix} dlnQ_1 \\ dlnQ_2 \\ dlnP_1 \\ dlnP_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \epsilon_{12}(\eta_{21} - \epsilon_{21}) + \epsilon_{11}(\epsilon_{22} - \eta_{22}) & \eta_{12}\epsilon_{11} - \eta_{11}\epsilon_{12} \\ \eta_{21}\epsilon_{22} - \eta_{22}\epsilon_{21} & \epsilon_{21}(\eta_{12} - \epsilon_{12}) + \epsilon_{22}(\epsilon_{11} - \eta_{11}) \\ \epsilon_{22} - \eta_{22} & \eta_{12} - \epsilon_{12} \\ \eta_{21} - \epsilon_{21} & \epsilon_{11} - \eta_{11} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$$+ \frac{1}{D} \begin{bmatrix} \eta_{12}(\epsilon_{21} - \eta_{21}) + \eta_{11}(\eta_{22} - \epsilon_{22}) & \eta_{11}\epsilon_{12} - \eta_{12}\epsilon_{11} \\ \eta_{22}\epsilon_{21} - \eta_{21}\epsilon_{22} & \eta_{22}(\eta_{11} - \epsilon_{11}) + (\eta_{21}(\epsilon_{12} - \eta_{12}) \\ \eta_{22} - \epsilon_{22} & \epsilon_{12} - \eta_{12} \\ \epsilon_{21} - \eta_{21} & \eta_{11} - \epsilon_{11} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

where $D = (\varepsilon_{11} - \eta_{11})(\varepsilon_{22} - \eta_{22}) - (\varepsilon_{12} - \eta_{12})(\varepsilon_{21} - \eta_{21})$

The Muth model is among the most famous and well-used models in agricultural economics. It comes from Richard F. Muth's 1964 paper titled "The Derived Demand Curve for a Productive Factor and the Industry Supply Curve".

The objective is to understand both the output and input markets.

We can use this similarly as the previous models to understand the effects of supply and demand shifters and various government policies.

The Muth model includes a demand function for the product and supply functions for the two factors, linked by a production function characterized by constant returns to scale and competitive market clearing.

Together these imply a derived supply function for the product and derived demand functions for the inputs.

EDMs – The Muth Model

$$
\begin{aligned}
Q^d &= D^d(P, a) \\
Q^s &= q(X_1, X_2, D, G) \\
W_1 &= Pq_1(X_1, X_2, D, G) \\
W_2 &= Pq_2(X_1, X_2, D, G) \\
X_1 &= g(W_1, b_1) \\
X_2 &= h(W_2, b_2)
\end{aligned}
$$

First differencing and transforming, we get:

$$
\begin{aligned}
d \ln Q^d &= \eta[d \ln P - \alpha] \\
d \ln Q^s &= s_1 \; d \ln X_1 + s_2 \; d \ln X_2 + \delta \\
d \ln W_1 &= d \ln P - \frac{s_2}{\sigma} d \ln X_1 + \frac{s_2}{\sigma} d \ln X_2 + \delta + \gamma \\
d \ln W_2 &= d \ln P + \frac{s_1}{\sigma} d \ln X_1 - \frac{s_1}{\sigma} d \ln X_2 + \delta - \frac{s_1}{s_2} \gamma \\
d \ln X_1 &= \varepsilon_1[d \ln W_1 + \beta_1] \\
d \ln X_2 &= \varepsilon_2[d \ln W_2 + \beta_2]
\end{aligned}
$$